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Hallucination Risk and Trustworthiness of Generative AI Systems Based on IVMPF-MABAC Decision-Making Strategies

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Abstract

Hallucination risk and trustworthiness of a generative artificial intelligence system play an important and vital role in daily-life scenarios. The terms Hallucination risk and trustworthiness are very famous for the generative artificial intelligence techniques to confirm that the final results or outputs are safe, accurate, and reliable for users or customers, particularly in education, finance, and healthcare. The main purpose of this manuscript is to evaluate the best and worst decisions among the decisions of Gemini (Google DeepMind), Copilot (GitHub/Microsoft), LLaMA (Meta), Claude (Anthropic), and ChatGPT (OpenAI) based on the following criteria, such as data privacy, explainability, Hallucination risk, reliability, and bias control. Therefore, the system of interval-valued m-polar fuzzy-multi-attribute border approximation area comparison technique for algebraic information is developed. The invented model is very famous and reliable for coping with awkward and complex information. Therefore, for the valuation or the construction of the derived technique, this study aims to develop the novel technique of interval-valued m-polar fuzzy information and also investigate their operations based on algebraic norms for the construction of the averaging operator and geometric operator. Lastly, this study also works on the analysis of the numerical examples and the implementation of the comparative analysis among proposed and prevailing frameworks to derive the validity and effectiveness of the introduced approaches.

Keywords:

Artificial intelligence; Decision-making techniques; Interval-valued m-polar fuzzy sets; MABAC models; Hallucination risks.

1. Introduction

This section briefly explains the overview of the existing techniques and ideas, especially about the Hallucination risk and trustworthiness of generative artificial intelligence systems. Furthermore, this section also works on the analysis of the fuzzy framework and its extension, which are very close to the proposed topic. Moreover, the literature review, research gap, motivation, main contribution, and summary of the proposed manuscript are part of this section.

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1.1. Hallucination Risk and Trustworthiness of a Generative Artificial Intelligence System

Artificial intelligence (AI) and machine learning techniques are playing a valuable and efficient role in coping with vague and uncertain information, but nowadays, the technique of Generative AI systems is a more powerful and useful technique that is widely used across multiple real-life applications; for instance, business analytics, healthcare, online decision support, and education. The generative AI systems learn from large data sets rather than direct reasoning and generate responses based on patterns learned. The information produced by these systems may sometimes be unsupported because of their nature and the condition of the data. The phenomenon or procedure is called hallucination, which directly affects the accuracy and reliability of generative AI systems. It also influences the trustworthiness of these systems. Trustworthiness not only relies on the correct information but also focuses on the accuracy, reliability, and transparency of the system. Trustworthy systems enhance the users' trust, providing accurate information and protecting their privacy. A trustworthy person also operates continuously across multiple tasks. Some key challenges related to trust and hallucination are briefly explained.

1) ***Hallucination:*** The generative AI systems generate some results, which may look correct, but in actuality, they are fabricated information. It directly affects the reliability and accuracy of the systems.
2) ***Bias:*** Sometimes the information produced by generative AI systems contains unfair or unbalanced responses, which also affects the system's efficiency.
3) ***Data Privacy:*** In generative AI systems, there is a risk of exposing sensitive or confidential information, which reduces the users' trust in the system.
4) ***Lack of Explainability:*** It is very difficult to understand how the outputs are generated because these systems generate outputs based on learned patterns.
5) ***Reliability:*** The generative AI systems have inconsistent performance across different fields, which directly affects the reliability of the system.

The evaluation of generative AI systems is not an easy task because trust and risk are subjective concepts in it. The evaluation approach of each user depends on their experience, and it may be different from one another even if they are evaluating the same system for a different purpose. All the existing evaluation approaches are limited because most of them depend on numerical accuracy metrics and are limited to representing human judgments. It requires advanced evaluation approaches because the information is more likely to be expressed in linguistic terms. So, these challenges make the evaluation process very complex and uncertain. The generative AI systems require a well-organized and accurate evaluation framework, which can effectively evaluate them. To understand the effectiveness and importance of systematic assessment, some key advantages are discussed, which clearly highlight the importance of a structured evaluation approach for generative AI systems.

1) System Improvement: A systematic evaluation approach enables the experts and developers to identify system weaknesses. It helps the system to operate continuously and saves time.
2) Decision Safety: An effective evaluation framework reduces the risk of incorrect information because it gives equal importance to each factor. It enhances the reliability of the system and also strengthens users’ trust.
3) Policy Support: A well-organized and effective evaluation approach assists regulators in AI governance. It also enhanced user trust and supports system reliability.

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4) Ethical AI: A systematic evaluation framework brings fairness to the systems and supports responsible AI use. This aspect strengthens the user's trust in the system.
5) User Trust: An effective evaluation framework increases confidence in generating AI outputs. It reduces the generation of inaccurate outputs.

1.2. Fuzzy Frameworks and Their Extensions

Decision-making frameworks are efficient for the assessment of the best and worst decisions among a collection of alternatives without criteria or with a single criterion. The decision-making assessment is one of the most reliable and efficient models for coping with uncertain and vague information in complex circumstances. The decision-making has become very challenging in modern management and engineering problems because of uncertain and incomplete information. The existing decision-making concepts are limited to representing human judgment, especially when the expert information is in subjective form rather than a numerical value. To handle this complexity, researchers invented modified mathematical tools for capturing vagueness and uncertainty. Among all, one of the most effective developments is fuzzy set (FS), which was introduced by Zadeh [1] in 1965. The FS describes such situations where the judgment cannot be expressed in partial rather than yes or no terms. The FS assigns a partial membership degree to each element of the set between 0 and 1. It captures the specific degree of an option to which it fully satisfies a specific condition using the membership function.

The truth value in a fuzzy framework represents the numerical value against each term from the universal set, but in many real-world scenarios, the real value is not enough to predict accurate and efficient results compared to a value in the shape of an interval. The FS effectively models the situation where the expert information is expressed in a single precise value. But in most real-world decision-making situations, experts prefer to express their preference as a range rather than a fixed point, which is a very serious problem in decision-making theory. To capture such situations, Gorzalczany [2] invented the notation of an interval-valued FS (IVFS) in 1987. In IVFS, the membership degree is represented by an interval instead of a single value, which provides a more reliable space for decision-makers to make a decision precisely. The IVFS provides a more reliable way to model human uncertainty in complex scenarios. The notation of IVFS is very effective for the expert to express complex and uncertain judgments. It gives an effective framework for problems where information is vague and uncertain.

The truth function represents just one value against each term from universal sets at one time, but in many genuine-life scenarios, the experts or decision-makers need more values to defend their thoughts and opinions in front of someone. Because the first value is dealt with like a presenter, and the other values are shown as support of the first value. The group of information has worth to defend their suggestions and theme. Therefore, Chen et al. [3] invented the notation of multi-polar fuzzy set (m-PFS) to capture more nuanced uncertainty in decision making. The m-PFS is the generalization of traditional FS, which assigns multiple membership values against each value from the universal set. Each term reflects a different opinion, which makes it effective for a complex decision environment. The concept of m-PFS is very useful for such real-world problems that involve multiple interactive factors, which cannot be captured by using the traditional fuzzy approaches.

1.3. Literature Review

The hallucination risk and trustworthiness involve complex and vague information; an effective and reliable decision-making framework is required. Various studies have been presented by scholars to address these challenges, such as Rathkopf [4], who invented the notation of corrosive hallucination to handle the epistemically troubling subclasses.

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Ciubotaru [5] highlighted the negative effects of hallucination on the accuracy and user trust in a digital learning environment. Lee et al. [6] briefly examined the risk of generative AI and explained how user behavior can help to overcome these challenges. Elsayed [7] briefly examined the impact of hallucination information on student learning outcomes in large language models. Ahn [8] extensively highlighted the limitations of large language models to evade hallucination and maintain reliability.

The FS framework has become a powerful foundation for modern decision-making and is widely used by researchers across different domains. Guiffrida and Nagi [9] presented a detailed literature review on the applications of FS in the product management research environment. Bloch and Ralescu [10] discussed various methods of FS in the imaging processing domain. Miliauskaitė and Kalibatiene [11] introduced a chronological and bibliometric analysis of FS based on existing papers extracted from Web of Science. Maues et al. [12] invented a fuzzy-based model to estimate construction waste generation. Guan [13] constructed a data management algorithm based on fuzzy information for handling the data of higher education. Budayan et al. [14] invented a computerized method using the concept of FS for delay risk assessment. Tolis et al. [15] merged a small greenhouse structure using the integrated approach of FS and Internet of Things. Abbate et al. [16] used the qualitative comparative analysis approach based on fuzzy information for the analysis of an Internet of Things platform. Krechko and Mikhaylov [17] used the concepts of FS and spatial analysis to generate global electricity from renewable sources.

Because of its effectiveness, the researchers used it across many fields, such as Kansal and Kumar [18] constructed a novel accuracy measure based on interval-valued fuzzy information for multi-criteria decision-making and recognizing patterns. Nithya et al. [19] invented the notation of digital weighted multi-criteria decision making based on interval-valued fuzzy information for the selection of the best contractor for building projects. Sanz et al. [20] used the integrated concept of IVFS and genetic amplitude tuning to enhance the performance of fuzzy rule-based classification systems. Wu et al. [21] constructed a triangular fuzzy rating model based on interval-valued fuzzy information for the representation of users' preferences. Petry and Yager [22] invented some aggregation and evaluation approaches based on interval-valued fuzzy information and discussed their applications in many areas. Ashtiani et al. [23] invented a novel concept of the TOPSIS model based on interval-valued fuzzy information and discussed its application in multi-criteria decision making.

Due to this, the concept of m-PFS was extended to different environments, such as Akram et al. [24] constructed two measures of connectedness under an m-polar fuzzy environment and extended its applications to the product manufacturing environment. Jagtap and Karande [25] constructed an m-polar fuzzy-based algorithm for the selection of a nontraditional machining process. Jagtap and Ghangale [26] constructed a novel multi-attribute decision-making approach based on m-polar fuzzy information for the selection of electric vehicles. Ali et al. [27] constructed some m-polar fuzzy-based aggregation operators for the analysis of multi-criteria decision making. Jagtap and Karande [28] presented a detailed review of the applications of m-PFS and also highlighted its benefits and flaws. Ali and Alsager [29] constructed the notation of power geometric aggregation operators under an m-polar fuzzy domain and extended their applications to urban transportation management. Akram et al. [30] deliberated the ELECTRE-I for m-PFSs. Further, Rahman et al. [31] derived the operators based on Aczel-Alsina norms for m-PFSs. Additionally, Jana and Hezam [32] presented the EDAS technique for m-PFSs.

Aggregation operators are widely used for the integration of multiple attribute values into a single overall assessment. Different types of fuzzy-based aggregation operators have been developed by researchers, such as Mezei and Wikstrom [33], who constructed a novel

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aggregation approach by integrating the notations of IVFS and ordered weighting averaging aggregation operators. Bustince et al. [34] deliberated the Choquet integrals for IVFSs. Further, Waseem et al. [35] derived the operators based on Hamacher norms for m-PFSs. Moreover, Akram et al. [36] examined the operators for m-PFSs based on Dombi norms. Additionally, Jana and Pal [37] presented the decision-making models for some m-PFSs. Further, Maity and Pal [38] stated the power operators based on Dombi norms for m-PFSs. Moreover, Ali et al. [39] derived the operators for m-PFSs based on Schweizer-Sklar norms. Many decision-making procedures are developed for the interpretation of complex problems. Inspired by these decision-making models, the hybrid decision-making model was invented by Pamucar and Cirovic [40], which is called the multi-attribute border approximation area comparison (MABAC) network. The MABAC model is very efficient in coping with complex data. Further, Akram et al. [41] derived another hybrid model of decision-making technique, called the PROMETHEE technique for m-PFSs. Moreover, Jagtap and Karande [42] invented the technique for order performance by similarity to the ideal solution (TOPSIS) model for m-PFSs. Additionally, Akram and Adeel [43] deliberated the ELECTRE-I and ELECTRE-II models for m-PFSs. Sahoo et al. [44] deliberated on sustainable agile manufacturing based on Pythagorean FSs. Ali et al. [45] invented the decision-making model for the multi-polar fuzzy soft expert technique. Sahoo and Debnath [46] exposed the AHP and MABAC model for a case study on pharmaceutical supplier selection. Ali et al. [47] presented the similarity measures for bipolar fuzzy soft information. Sahoo and Debnath [48] stated the fuzzy decision-making model for supporting resource-efficient manufacturing techniques. Sarwar et al. [49] designed the hesitant 2-tuple linguistic rough numbers. Liu,et.al [50] studied lung diseases recognition using generalized complex fermatean fuzzy distance and entropy measures. Liu, et.al [51] investigated elementary function-based fermatean fuzzy similarity measures using decision making tools. Khan, et.al [52] studied covering-based intuitionistic hesitant fuzzy rough set models and their application to decision-making problems.

1.4. Main Purpose of Proposed Manuscript

This section aims to deliberate on the research gap, main problem, advantages, motivation, and main contribution of the derived models. The main theme of this manuscript is to develop the interval-valued multi-polar fuzzy (IVMPF)-MABAC model based on aggregation information. Many scholars widely used the aggregation operators to aggregate complex and uncertain expert information based on fuzzy information, but not IVMPF data sets. Similarly, they applied various MADM models to evaluate and rank alternatives in an uncertain environment, but not hybrid models. All these existing aggregation operators and MADM models effectively handle various types of complex and uncertain real-world problems. Despite its effectiveness, there are still some huge research gaps in the existing literature, such as: (i) The existing literature contains many frameworks, which help the experts to express their preference, but an effective framework of IVMPF is not offered yet. (ii) Further, this study also discusses the different aggregation operators, where the technique of IVMPF-based aggregation operators has not been developed yet. (iii) Different generalized ideas of the MABAC model have been developed, but no one has invented an effective idea of the MABAC model under an IVMPF environment. (iv) The current literature contains various hybrid ideas, but no one integrates the notation of the MABAC model with aggregation operators under an IVMPF environment. The proposed research work is highly motivated by these research gaps. Therefore, this study aims to build such ideas that effectively cover all these research gaps and enhance decision-making approaches. This study aims to build the following concepts:

1) This study aims to invent an advanced framework of IVMPF, which enables the expert to express their preference even if it is complex.

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2) This study also aims to define some novel aggregation operators based on IVMPF information, which helps the decision-makers to handle complex information.
3) Our target is to extend the idea of the MABAC model to an IVMPF environment.
4) This study also aims to integrate the notion of the MABAC model with aggregation operators under an IVMPF environment.

The presented idea of an IVMPF framework integrates the strengths of individual ideas, which makes it very reliable for modelling complex and uncertain expert preferences. The effectiveness and importance of the proposed idea can be understood from the following advantages.

1) The proposed notation of the IVMPF framework is very effective for the expert to express their preference.
2) The IVMPF-based aggregation operators effectively aggregated complex and uncertain expert information.
3) The generalized notation of the MABAC model enables the decision-makers to accurately evaluate and rank complex alternatives having multiple attributes.
4) The framework of the MABAC model, based on IVMPF information, is very flexible and can be extended to different environments such as sustainability, healthcare, supply chain management, and information management.

The notion of IVMPF set (IVMPFS) is a generalization of some existing ideas, and all these ideas have now become special cases of the IVMPFS; for instance, FS, IVFS, and m-PFS. Similarly, the proposed aggregation operators (AOs) and MADM model are also generalizations of many existing ideas, and all of them have now become special cases of the derived approaches; for instance,

1) The arithmetic AOs.
2) The geometric AOs.
3) The weighted averaging AOs.
4) The weighted geometric AOs.
5) The MABAC model is based on fuzzy information.

The limitations of the existing models and the benefits of the proposed models show that the construction of the new model is very important because of ambiguity and problems. Therefore, this study works on the following valuation, such as

1) To invent the novel idea of IVMPFS and also explain its basic properties.
2) To propose some novel aggregation operators using the IVMPF concept.
3) To construct a generalized idea of the MABAC model under an IVMPF environment.
4) To address a real-world problem and solve it with the help of the proposed model.
5) To perform the sensitivity analysis and comparative analysis.

This manuscript is constructed in the following form: for instance, Section 2 briefly discusses the information of FSs, IVFSs, and m-PFSs. In Section 3, the technique of the IVMPF-MABAC model is developed. In Section 4, the application of Hallucination risk and trustworthiness of generative artificial intelligence systems is performed based on some numerical examples to describe the validity of the derived theory. In Section 5, the comparative analysis and sensitive analysis are evaluated for the derived examples. Finally, we discussed some concluding remarks in Section 6. The geometrical framework for the proposed manuscript is deliberated in Figure 1.

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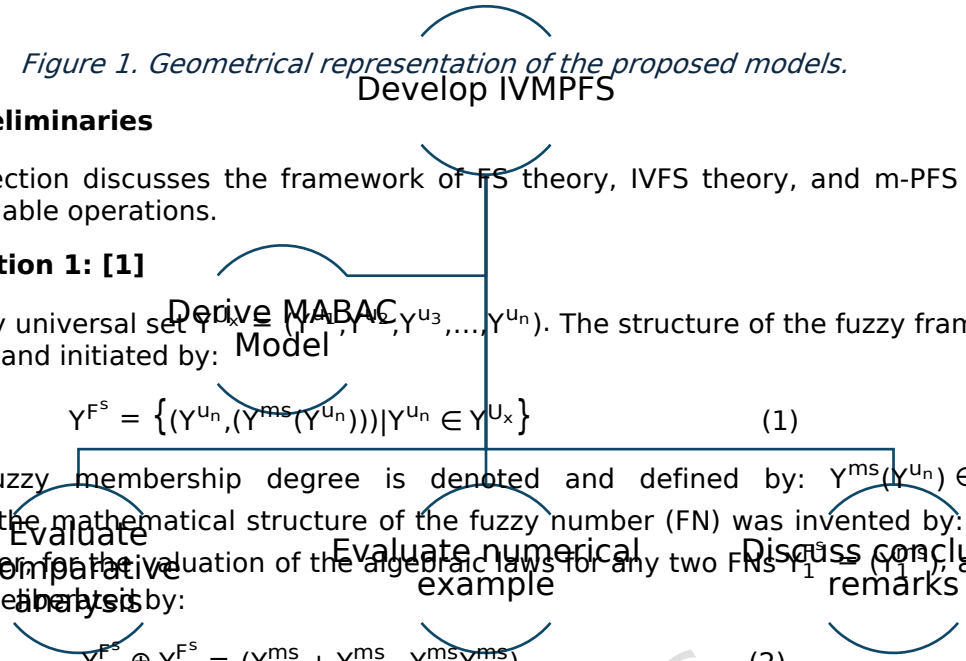


Figure 1. Geometrical representation of the proposed models.

2. Preliminaries

This section discusses the framework of FS theory, IVFS theory, and m-PFS theory with their valuable operations.

Definition 1: [1]

For any universal set $Y^{U_x} = (Y^{u_1},Y^{u_2},Y^{u_3},...,Y^{u_n})$. The structure of the fuzzy framework is deliberated and initiated by:

$$Y^{F^s} = \left\{(Y^{u_n},(Y^{ms}(Y^{u_n})))|Y^{u_n} \in Y^{U_x}\right\} \quad (1)$$

The fuzzy membership degree is denoted and defined by: $Y^{ms}(Y^{u_n}) \in [0,1]$. Furthermore, the mathematical structure of the fuzzy number (FN) was invented by: $Y_N^{F^s} = (Y_N^{ms})$. Moreover, for the valuation of the algebraic laws for any two FNs $Y_1^{F^s} = (Y_1^{ms})$, and $Y_2^{F^s} = (Y_2^{ms})$ are deliberated by:

$$Y_1^{F^s} \oplus Y_2^{F^s} = (Y_1^{ms} + Y_2^{ms} - Y_1^{ms}Y_2^{ms}) \quad (2)$$

$$Y_1^{F^s} \otimes Y_2^{F^s} = (Y_1^{ms}Y_2^{ms}) \quad (3)$$

$$\lambda Y_1^{F^s} = \left(1 - (1 - Y_1^{ms})^{\lambda}\right) \quad (4)$$

$$\left(Y_1^{F^s}\right)^{\lambda} = \left((Y_1^{ms})^{\lambda}\right) \quad (5)$$

Definition 2: [2]

For any universal set $Y^{U_x} = (Y^{u_1},Y^{u_2},Y^{u_3},...,Y^{u_n})$. The structure of the interval-valued fuzzy framework is deliberated and initiated by:

$$Y^{I_v} = \left\{\left(Y^{u_n},\left[Y^{ms^l}(Y^{u_n}),Y^{ms^u}(Y^{u_n})\right]\right)|Y^{u_n} \in Y^{U_x}\right\} \quad (6)$$

The interval-valued fuzzy membership degree is denoted and defined by: $\left[Y^{ms^l}(Y^{u_n}),Y^{ms^u}(Y^{u_n})\right] \subseteq [0,1]$. Furthermore, the mathematical structure of the interval-valued fuzzy number (IVFN) was invented by: $Y_N^{F^s} = \left(\left[Y^{ms^l}(Y^{u_n}),Y^{ms^u}(Y^{u_n})\right]\right)$.

Definition 3: [3]

For any universal set $Y^{U_x} = (Y^{u_1},Y^{u_2},Y^{u_3},...,Y^{u_n})$. The structure of the multi-polar fuzzy framework is deliberated and initiated by:

$$Y^{M_P} = \left\{(Y^{u_n},(Y^{ms_1}(Y^{u_n}),Y^{ms_2}(Y^{u_n}),...,Y^{ms_m}(Y^{u_n})))|Y^{u_n} \in Y^{U_x}\right\} \quad (7)$$

The multi-fuzzy membership degree is denoted and defined by: $(Y^{ms_1}(Y^{u_n}),Y^{ms_2}(Y^{u_n}),...,Y^{ms_m}(Y^{u_n})) \in [0,1]$. Furthermore, the mathematical structure of the multi-polar fuzzy number (m-PFN) was invented by: $Y_N^{M_P} = \left(Y_N^{ms_1},Y_N^{ms_2},...,Y_N^{ms_m}\right)$. Moreover, for the valuation of the algebraic laws for any two m-PFNs $Y_1^{M_P} = \left(Y_1^{ms_1},Y_1^{ms_2},...,Y_1^{ms_m}\right)$, and $Y_2^{M_P} = \left(Y_2^{ms_1},Y_2^{ms_2},...,Y_2^{ms_m}\right)$ are deliberated by:

$$Y_1^{M_P} \oplus Y_2^{M_P} = \left(Y_1^{ms_1} + Y_2^{ms_1} - Y_1^{ms_1}Y_2^{ms_1},Y_1^{ms_2} + Y_2^{ms_2} - Y_1^{ms_2}Y_2^{ms_2},...,Y_1^{ms_m} + Y_2^{ms_m} - Y_1^{ms_m}Y_2^{ms_m}\right) \quad (8$$

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$$Y_1^{M_P} \otimes Y_2^{M_P} = \left(Y_1^{ms_1}Y_2^{ms_1}, Y_1^{ms_2}Y_2^{ms_2}, \ldots, Y_1^{ms_m}Y_2^{ms_m}\right) \quad (9)$$

$$\lambda Y_1^{M_P} = \left(1 - \left(1 - Y_1^{ms_1}\right)^{\lambda}, 1 - \left(1 - Y_1^{ms_2}\right)^{\lambda}, \ldots, 1 - \left(1 - Y_1^{ms_m}\right)^{\lambda}\right) \quad (10)$$

$$\left(Y_1^{M_P}\right)^{\lambda} = \left(\left(Y_1^{ms_1}\right)^{\lambda}, \left(Y_1^{ms_2}\right)^{\lambda}, \ldots, \left(Y_1^{ms_m}\right)^{\lambda}\right) \quad (11)$$

3. IVMPF-MABAC Decision-Making Strategies

This section is famous for the valuation of the IVMPFS and their operations based on algebraic norms. Further, using the proposed information, this study also developed the aggregation operators for the construction of the MABAC models.

3.1. IVMPFS: Interval-Valued Multi-Polar Fuzzy Set

This section only develops the novel system of IV-MPFSs and their valuable operations.

Definition 4:

For any universal set $Y^{U_x} = (Y^{u_1}, Y^{u_2}, Y^{u_3}, \ldots, Y^{u_n})$. The structure of the interval-valued multi-polar fuzzy framework is deliberated and initiated by:

$$Y^{I_m} = \left\{\left(Y^{u_n}, \left(\left[Y^{ms_1^l}(Y^{u_n}), Y^{ms_1^u}(Y^{u_n})\right], \left[Y^{ms_2^l}(Y^{u_n}), Y^{ms_2^u}(Y^{u_n})\right], \ldots, \left[Y^{ms_m^l}(Y^{u_n}), Y^{ms_m^u}(Y^{u_n})\right]\right)\right) | Y^{u_n} \in Y^{U_x}\right\} \quad (12)$$

Where $\left[Y^{ms_i^l}(Y^{u_n}), Y^{ms_i^u}(Y^{u_n})\right]$ shows a family of fuzzy closed intervals, which represent the membership function for $i = (1,2,3,\ldots,m)$, such as $\left[Y^{ms_i^l}(Y^{u_n}), Y^{ms_i^u}(Y^{u_n})\right] \subseteq [0,1]$. The IVMPF number (IVMPFN) is shown by: $Y_N^{I_m} = \left(\left[Y_N^{ms_1^l}, Y_N^{ms_1^u}\right], \left[Y_N^{ms_2^l}, Y_N^{ms_2^u}\right], \ldots, \left[Y_N^{ms_m^l}, Y_N^{ms_m^u}\right]\right)$. Moreover, for the valuation of the algebraic laws for any two IVMPFNs $Y_1^{I_m} = \left(\left[Y_1^{ms_1^l}, Y_1^{ms_1^u}\right], \left[Y_1^{ms_2^l}, Y_1^{ms_2^u}\right], \ldots, \left[Y_1^{ms_m^l}, Y_1^{ms_m^u}\right]\right)$, and $Y_2^{I_m} = \left(\left[Y_2^{ms_1^l}, Y_2^{ms_1^u}\right], \left[Y_2^{ms_2^l}, Y_2^{ms_2^u}\right], \ldots, \left[Y_2^{ms_m^l}, Y_2^{ms_m^u}\right]\right)$ are deliberated by:

$$Y_1^{I_m} \oplus Y_2^{I_m} = \left\langle \begin{matrix} \left[Y_1^{ms_1^l} + Y_2^{ms_1^l} - Y_1^{ms_1^l}Y_2^{ms_1^l}, Y_1^{ms_1^u} + Y_2^{ms_1^u} - Y_1^{ms_1^u}Y_2^{ms_1^u}\right], \\ \left[Y_1^{ms_2^l} + Y_2^{ms_2^l} - Y_1^{ms_2^l}Y_2^{ms_2^l}, Y_1^{ms_2^u} + Y_2^{ms_2^u} - Y_1^{ms_2^u}Y_2^{ms_2^u}\right], \ldots, \\ \left[Y_1^{ms_m^l} + Y_2^{ms_m^l} - Y_1^{ms_m^l}Y_2^{ms_m^l}, Y_1^{ms_m^u} + Y_2^{ms_m^u} - Y_1^{ms_m^u}Y_2^{ms_m^u}\right] \end{matrix} \right\rangle \quad (13)$$

$$Y_1^{I_m} \otimes Y_2^{I_m} = \left\langle \begin{matrix} \left[Y_1^{ms_1^l}Y_2^{ms_1^l}, Y_1^{ms_1^u}Y_2^{ms_1^u}\right], \left[Y_1^{ms_2^l}Y_2^{ms_2^l}, Y_1^{ms_2^u}Y_2^{ms_2^u}\right], \\ \left[Y_1^{ms_m^l}Y_2^{ms_m^l}, Y_1^{ms_m^u}Y_2^{ms_m^u}\right] \end{matrix} \right\rangle \quad (14)$$

$$\lambda Y_1^{I_m} = \left\langle \begin{matrix} \left[1 - \left(1 - Y_1^{ms_1^l}\right)^{\lambda}, 1 - \left(1 - Y_1^{ms_1^u}\right)^{\lambda}\right], \left[1 - \left(1 - Y_1^{ms_2^l}\right)^{\lambda}, 1 - \left(1 - Y_1^{ms_2^u}\right)^{\lambda}\right], \ldots, \\ \left[1 - \left(1 - Y_1^{ms_m^l}\right)^{\lambda}, 1 - \left(1 - Y_1^{ms_m^u}\right)^{\lambda}\right] \end{matrix} \right\rangle \quad (15)$$

$$\left(Y_1^{I_m}\right)^{\lambda} = \left(\left[\left(Y_1^{ms_1^l}\right)^{\lambda}, \left(Y_1^{ms_1^u}\right)^{\lambda}\right], \left[\left(Y_1^{ms_2^l}\right)^{\lambda}, \left(Y_1^{ms_2^u}\right)^{\lambda}\right], \ldots, \left[\left(Y_1^{ms_m^l}\right)^{\lambda}, \left(Y_1^{ms_m^u}\right)^{\lambda}\right]\right) \quad (16)$$

Definition 5:

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Let $Y_N^{l_m} = \left(\left[Y_N^{ms_1^l}, Y_N^{ms_1^u}\right], \left[Y_N^{ms_2^l}, Y_N^{ms_2^u}\right], \ldots, \left[Y_N^{ms_m^l}, Y_N^{ms_m^u}\right]\right)$ be any IVMPFN. Then

$$Y^{S^f}\left(Y_N^{l_m}\right) = \frac{1}{2m}\left(\sum_{i=1}^{m} Y_N^{ms_i^l} + \sum_{i=1}^{m} Y_N^{ms_i^u}\right) \tag{17}$$

The function $Y^{S^f}\left(Y_N^{l_m}\right)$, is called the score function.

3.2. Aggregation Operators for IVMPFNs

This section develops the model of the IVMPF weighted averaging (IVMPFWA) operator and the IVMPF weighted geometric (IVMPFWA) operator by using the invented operations.

Definition 6:

For any family of IVMPFNs $Y_N^{l_{mk}} = \left(\left[Y_N^{ms_{1_k}^l}, Y_N^{ms_{1_k}^u}\right], \left[Y_N^{ms_{2_k}^l}, Y_N^{ms_{2_k}^u}\right], \ldots, \left[Y_N^{ms_{m_k}^l}, Y_N^{ms_{m_k}^u}\right]\right)$, i = (1,2,3,...,m) with weight vectors $\sum_{k=1}^{n} Y_{W(i)} = 1$. The IVMPFWA operator is deliberated and initiated by:

$$\text{IVMPFWA}\left(Y_N^{l_{m1}}, Y_N^{l_{m2}}, Y_N^{l_{m3}}, \ldots, Y_N^{l_{mn}}\right) = \oplus_{k=1}^{n}\left(Y_{W(k)} Y_N^{l_{mk}}\right) \tag{18}$$

Thus, using the proposed operational laws, we have

$$\begin{aligned}&\text{IVMPFWA}\left(Y_N^{l_{m1}}, Y_N^{l_{m2}}, Y_N^{l_{m3}}, \ldots, Y_N^{l_{mn}}\right)\\ &= \left(\begin{array}{c}\left[1 - \prod_{k=1}^{n}\left(1 - Y_N^{ms_{1_k}^l}\right)^{Y_{W(k)}}, 1 - \prod_{k=1}^{n}\left(1 - Y_N^{ms_{1_k}^u}\right)^{Y_{W(k)}}\right],\\ \left[1 - \prod_{k=1}^{n}\left(1 - Y_N^{ms_{2_k}^l}\right)^{Y_{W(k)}}, 1 - \prod_{k=1}^{n}\left(1 - Y_N^{ms_{2_k}^u}\right)^{Y_{W(k)}}\right], \ldots,\\ \left[1 - \prod_{k=1}^{n}\left(1 - Y_N^{ms_{m_k}^l}\right)^{Y_{W(k)}}, 1 - \prod_{k=1}^{n}\left(1 - Y_N^{ms_{m_k}^u}\right)^{Y_{W(k)}}\right]\end{array}\right)\end{aligned} \tag{19}$$

Definition 7:

For any family of IVMPFNs $Y_N^{l_{mk}} = \left(\left[Y_N^{ms_{1_k}^l}, Y_N^{ms_{1_k}^u}\right], \left[Y_N^{ms_{2_k}^l}, Y_N^{ms_{2_k}^u}\right], \ldots, \left[Y_N^{ms_{m_k}^l}, Y_N^{ms_{m_k}^u}\right]\right)$, i = (1,2,3,...,m) with weight vectors $\sum_{k=1}^{n} Y_{W(i)} = 1$. The IVMPFWG operator is deliberated and initiated by:

$$\text{IVMPFWG}\left(Y_N^{l_{m1}}, Y_N^{l_{m2}}, Y_N^{l_{m3}}, \ldots, Y_N^{l_{mn}}\right) = \otimes_{k=1}^{n}\left(Y_N^{l_{mk}}\right)^{Y_{W(k)}} \tag{20}$$

Thus, using the proposed operational laws, we have

$$\begin{aligned}&\text{IVMPFWG}\left(Y_N^{l_{m1}}, Y_N^{l_{m2}}, Y_N^{l_{m3}}, \ldots, Y_N^{l_{mn}}\right)\\ &= \left(\begin{array}{c}\left[\prod_{k=1}^{n}\left(Y_N^{ms_{1_k}^l}\right)^{Y_{W(k)}}, \prod_{k=1}^{n}\left(Y_N^{ms_{1_k}^u}\right)^{Y_{W(k)}}\right], \left[\prod_{k=1}^{n}\left(Y_N^{ms_{2_k}^l}\right)^{Y_{W(k)}}, \prod_{k=1}^{n}\left(Y_N^{ms_{2_k}^u}\right)^{Y_{W(k)}}\right], \ldots,\\ \left[\prod_{k=1}^{n}\left(Y_N^{ms_{m_k}^l}\right)^{Y_{W(k)}}, \prod_{k=1}^{n}\left(Y_N^{ms_{m_k}^u}\right)^{Y_{W(k)}}\right]\end{array}\right)\end{aligned} \tag{21}$$

3.3. IVMPF-MABAC Model

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This section concentrates on the development of the MABAC model based on IVMPFS, called the IVMPF-MABAC technique, which is a hybrid decision-making model and used for the valuation of the best and worst decisions. This modifies the existing concept of the MABAC model and develops a novel notation of the MABAC model under an IVMPF environment. The proposed MABAC model can easily evaluate complex and uncertain expert information and is very effective for handling real-world problems. It covers all the limitations faced by the traditional models. It serves as a strong mathematical tool that enables decision makers to evaluate and rank complex expert judgment. The operational mechanism of the proposed model is explained below:

Step 1:

Construct a decision matrix from the expert judgements shown in an IVMPF form. The decision matrix is developed in the following shape.

$$Y_{D(M)} = [Y_{a_{ik}}]_{m\times n} = \begin{bmatrix} Y_{a_{11}} & Y_{a_{12}} & . & . & Y_{a_{1n}} \\ Y_{a_{21}} & Y_{a_{22}} & . & . & Y_{a_{2n}} \\ . & . & . & . & . \\ Y_{a_{m1}} & Y_{a_{m2}} & . & . & Y_{a_{mn}} \end{bmatrix} \tag{22}$$

Step 2:

Normalize the given expert judgement using the normalization concept. The following formula is used for normalizing the expert information.

$$Y_{ND(M)} = \begin{cases} \left(\left[Y_N^{ms^l_{1_k}}, Y_N^{ms^u_{1_k}}\right], \left[Y_N^{ms^l_{2_k}}, Y_N^{ms^u_{2_k}}\right], \ldots, \left[Y_N^{ms^l_{m_k}}, Y_N^{ms^u_{m_k}}\right]\right) & \text{BT} \\ \left(\left[Y_N^{ms^l_{1_k}}, Y_N^{ms^u_{1_k}}\right], \left[Y_N^{ms^l_{2_k}}, Y_N^{ms^u_{2_k}}\right], \ldots, \left[Y_N^{ms^l_{m_k}}, Y_N^{ms^u_{m_k}}\right]\right)' & \text{CT} \end{cases} \tag{23}$$

Where, $\left[Y_N^{ms^l_{1_k}}, Y_N^{ms^u_{1_k}}\right]' = \left[1 - Y_N^{ms^u_{1_k}}, 1 - Y_N^{ms^l_{1_k}}\right]$.

Step 3:

Calculate a weighted normalized decision matrix from the information located in Step 2. The following formulas are used for finding the weighted normalized decision matrix, which is available in Eq. (15) and Eq. (16).

Step 4:

Determine the border approximation area from the information located in Step 3. The following formulas are used for determining the border approximation area is available in Eq. (19) and Eq. (21).

Step 5:

Calculate the distance of the alternatives from the pre-defined border approximation area using Step 4 information. The following formula is used for finding the distance between the alternatives.

$$Y_{D_{ik}} = \begin{cases} d\left(Y_N^{l_{mk}}, Y_N^{l_{mi}}\right), & \text{If } Y_N^{l_{mk}} > Y_N^{l_{mi}} \\ 0 & \text{If } Y_N^{l_{mk}} = Y_N^{l_{mi}} \\ -d\left(Y_N^{l_{mk}}, Y_N^{l_{mi}}\right), & \text{If } Y_N^{l_{mk}} < Y_N^{l_{mi}} \end{cases} \tag{24}$$

Where,

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$$d\left(Y_N^{l_{mk}}, Y_N^{l_{mi}}\right) = \frac{1}{2m}\left(\left|Y_N^{ms^l_{1_k}} - Y_N^{ms^l_{1_i}}\right| + \left|Y_N^{ms^u_{1_k}} - Y_N^{ms^u_{1_i}}\right| + \left|Y_N^{ms^l_{2_k}} - Y_N^{ms^l_{2_i}}\right| + \left|Y_N^{ms^u_{2_k}} - Y_N^{ms^u_{2_i}}\right| + \ldots + \left|Y_N^{ms^l_{m_k}} - Y_N^{ms^l_{m_i}}\right| + \left|Y_N^{ms^u_{m_k}} - Y_N^{ms^u_{m_i}}\right|\right) \quad (25)$$

Step 6:

Determine score values of each alternative using the information located in Step 5. The following formula is used for finding the score values of the alternatives.

$$Y^{S^f}\left(Y_N^{l_m}\right) = \frac{1}{n}\sum_{i,k=1}^{n} d\left(Y_N^{l_{mk}}, Y_N^{l_{mi}}\right) \quad (26)$$

Step 7:

Rank the alternatives according to their score values aggregated in Step 6. This final ranking will help the experts to get the best alternative from the given choices. Moreover, this study works on the valuation or assessment of the proposed examples to derive the validity and effectiveness of the derived approaches. The geometrical interpretation of the MABAC model is derived in Figure 2.

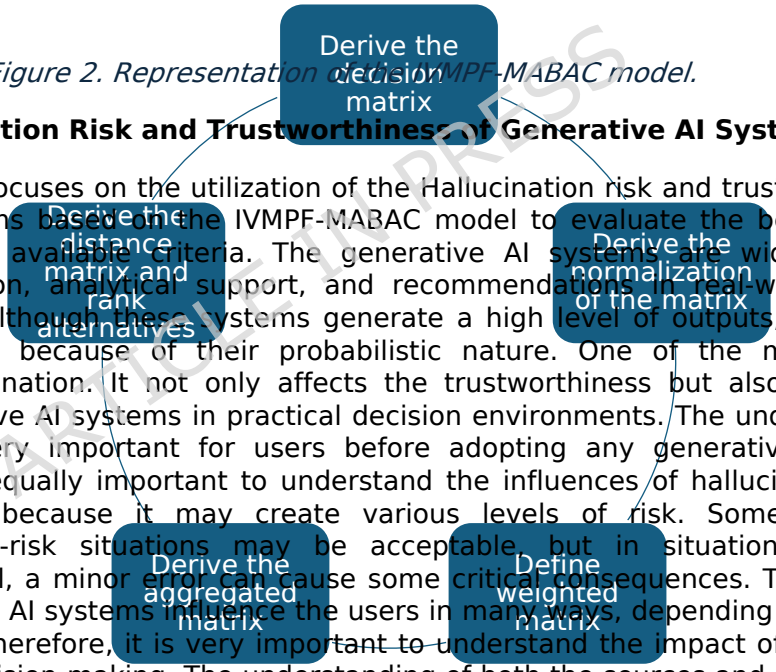


Figure 2. Representation of the IVMPF-MABAC model.

4. Hallucination Risk and Trustworthiness of Generative AI Systems

This section focuses on the utilization of the Hallucination risk and trustworthiness of generative AI systems based on the IVMPF-MABAC model to evaluate the best and worst decisions based on available criteria. The generative AI systems are widely used for producing information, analytical support, and recommendations in real-world decision-making situations. Although these systems generate a high level of outputs, and it's also produce uncertainty because of their probabilistic nature. One of the most common challenges is hallucination. It not only affects the trustworthiness but also reduces the adoption of generative AI systems in practical decision environments. The understanding of these sources is very important for users before adopting any generative AI system. Similarly, it is also equally important to understand the influences of hallucination on the decision outcomes because it may create various levels of risk. Sometimes, minor inaccuracies in low-risk situations may be acceptable, but in situations where the information is critical, a minor error can cause some critical consequences. The inaccurate outputs of generated AI systems influence the users in many ways, depending on the nature of the application. Therefore, it is very important to understand the impact of hallucination on the quality of decision-making. The understanding of both the sources and impact is very effective for the expert before adopting generative AI systems. As the origination of hallucination depends on many factors, the conventional evaluation approaches are limited in handling them. These factors contain subjective and uncertain information, requires well-structured and reliable decision-making approach. To conduct a reliable assessment of different AI systems, we select a set of well-known and widely used generative AI alternatives. The alternatives are briefly explained below:

1) **Gemini (Google DeepMind)**: The Gemini is a novel generative AI model invented by Google DeepMind. It effectively handles different types of data, including images, text, and more. The key advantage of this alternative is that it uses advanced technologies to understand the information provided by the users, which also helps to reduce errors. It

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overcomes the limitations faced by the traditional language models. Another effectiveness of this alternative is that its nature is multimodal, which makes it effective for better decision support. It integrates creativity with responsibility, which helps to minimize hallucinations.

2) **Copilot (GitHub/Microsoft)**: The Copilot is particularly designed for coding and software development. The programmers used this approach to automate repetitive tasks. The key advantage of Copilot is that it deeply focuses on the information while generating output. It also enhances productivity and is very effective for programming. The limitation of Copilot is that it is less effective for domains other than programming, and it also requires caution to avoid bugs. These limitations affect the adoption of Copilot.
3) **LLaMA (Meta):** The LLaMA is an effective large language model constructed by Meta. It provides an open space for researchers and developers. The trustworthiness of LLaMA totally depends on how the user trains and applies it. The key advantage of this model is that it enables the organization to reduce the hallucination. It is very effective for experimental and research purposes. However, it requires careful management and proper safeguards to ensure reliability.
4) **Claude (Anthropic):** Claude is an effective generative AI approach, invented by Anthropic. It is particularly designed to reduce biased, hallucinated, or harmful content in its response. The key advantage of this approach is that it avoids producing incorrect information whenever possible. This approach is very effective for situations where trustworthiness is a top priority. It reduces the risk of hallucination. It also enhanced the user's confidence and reduced errors crucially.
5) **ChatGPT (OpenAI):** ChatGPT is a well-known generative AI system widely used for generating natural and fluent conversations. It is very flexible and is used in domains such as content creation, education, and customer support. The key advantage of this approach is that it is very effective for performing general tasks. However, sometimes ChatGPT gives incorrect information and also does not provide any explanation about the final output, which may affect user trust. It also requires caution in critical risk applications.

To evaluate the performance and trustworthiness of the alternatives, we select a set of attributes. These attributes will help us to effectively evaluate and rank them by using the proposed MABAC model under an interval-valued m-polar fuzzy environment. The list of selected attributes is given as:

1) **Data Privacy:** Data privacy is a procedure or process that is used for the protection of sensitive and personal data from unauthorized systems or persons.
2) **Explainability**: Explainability is a very famous term that is used to describe how well a system or process clearly explains how and why it made a decision, or how you can understand the reasoning behind the results.
3) **Hallucination Risk:** Hallucination risk is basically representing the possibility that the artificial intelligence system generates data whose sound correct, but in actuality, it must be false.

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4) **Reliability**: Reliability means how dependably and consistently a system performs over time.
5) **Bias Control**: The bias control is a procedure that is used for the identification, prevention, and reeducation of unfairness bias.

The selection of these five attributes plays an important role in the selection of the alternatives among the five considered alternatives. Therefore, this study aims to use the procedure of the IVMPF-MABAC technique. Thus, using the procedure of the derived model, this study aims to evaluate the considered problems, such as

Step 1:

The decision matrix is arranged and constructed by using the data of IVMPF values in Table 1, which represent the five attributes for five alternatives.

Table 1. IVMPF decision matrix.

MABAC	$Y^{A_{t(1)}}$	$Y^{A_{t(2)}}$	$Y^{A_{t(3)}}$	$Y^{A_{t(4)}}$	$Y^{A_{t(5)}}$
$Y^{A'_{t(1)}}$	⟨[0.2,0.3],[0.1,0.5],[0.4,0.7]⟩	⟨[0.3,0.4],[0.2,0.7],[0.1,0.6]⟩	⟨[0.5,0.6],[0.4,0.7],[0.2,0.3]⟩	⟨[0.3,0.7],[0.4,0.5],[0.1,0.8]⟩	⟨[0.3,0.6],[0.2,0.5],[0.4,0.5]⟩
$Y^{A'_{t(2)}}$	⟨[0.21,0.31],[0.11,0.51],[0.41,0.71]⟩	⟨[0.31,0.41],[0.21,0.71],[0.11,0.61]⟩	⟨[0.51,0.61],[0.41,0.71],[0.21,0.31]⟩	⟨[0.31,0.71],[0.41,0.51],[0.11,0.81]⟩	⟨[0.31,0.61],[0.21,0.51],[0.41,0.51]⟩
$Y^{A'_{t(3)}}$	⟨[0.22,0.32],[0.12,0.52],[0.42,0.72]⟩	⟨[0.32,0.42],[0.22,0.72],[0.12,0.62]⟩	⟨[0.52,0.62],[0.42,0.72],[0.22,0.32]⟩	⟨[0.32,0.72],[0.42,0.52],[0.12,0.82]⟩	⟨[0.32,0.62],[0.22,0.52],[0.42,0.52]⟩
$Y^{A'_{t(4)}}$	⟨[0.23,0.33],[0.13,0.53],[0.43,0.73]⟩	⟨[0.33,0.43],[0.23,0.73],[0.13,0.63]⟩	⟨[0.53,0.63],[0.43,0.73],[0.23,0.33]⟩	⟨[0.33,0.73],[0.43,0.53],[0.13,0.83]⟩	⟨[0.33,0.63],[0.23,0.53],[0.43,0.53]⟩
$Y^{A'_{t(5)}}$	⟨[0.24,0.34],[0.14,0.54],[0.44,0.74]⟩	⟨[0.34,0.44],[0.24,0.74],[0.14,0.64]⟩	⟨[0.54,0.64],[0.44,0.74],[0.24,0.34]⟩	⟨[0.34,0.74],[0.44,0.54],[0.14,0.84]⟩	⟨[0.34,0.64],[0.24,0.54],[0.44,0.54]⟩

Step 2:

Actually, the data in Table 1 contained the positive types of information or benefit types of data without cost. Therefore, the considered data does not need to be normalized.

Step 3:

Consider the information in Eq. (15) for $\lambda = 0.5$, the data in Table 1 is converted to the data in Table 2, which is called a weighted matrix.

Table 2. The weighted normalized decision matrix.

MABAC	$Y^{A_{t(1)}}$	$Y^{A_{t(2)}}$	$Y^{A_{t(3)}}$	$Y^{A_{t(4)}}$	$Y^{A_{t(5)}}$
$Y^{A'_{t(1)}}$	⟨[0.106,0.163],[0.051,0.293],[0.225,0.452]⟩	⟨[0.163,0.225],[0.106,0.452],[0.051,0.368]⟩	⟨[0.293,0.368],[0.225,0.452],[0.106,0.163]⟩	⟨[0.163,0.452],[0.225,0.293],[0.051,0.553]⟩	⟨[0.163,0.368],[0.106,0.293],[0.225,0.293]⟩
$Y^{A'_{t(2)}}$	⟨[0.111,0.169],[0.057,0.300],[0.232,0.462]⟩	⟨[0.169,0.232],[0.111,0.462],[0.057,0.376]⟩	⟨[0.300,0.376],[0.232,0.462],[0.111,0.169]⟩	⟨[0.169,0.461],[0.232,0.300],[0.057,0.564]⟩	⟨[0.169,0.376],[0.111,0.300],[0.232,0.300]⟩
$Y^{A'_{t(3)}}$	⟨[0.117,0.175],[0.062,0.307],[0.238,0.471]⟩	⟨[0.175,0.238],[0.117,0.471],[0.062,0.384]⟩	⟨[0.307,0.384],[0.238,0.471],[0.117,0.175]⟩	⟨[0.175,0.471],[0.228,0.307],[0.062,0.576]⟩	⟨[0.175,0.384],[0.117,0.307],[0.238,0.307]⟩

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$Y^{A'_{t(4)}}$	⟨[0.123,0.181], [0.067,0.314], [0.245,0.480]⟩	⟨[0.181,0.245], [0.123,0.480], [0.067,0.392]⟩	⟨[0.314,0.392], [0.245,0.480], [0.123,0.181]⟩	⟨[0.181,0.480], [0.245,0.314], [0.067,0.588]⟩	⟨[0.181,0.392], [0.123,0.314], [0.245,0.314]⟩
$Y^{A'_{t(5)}}$	⟨[0.128,0.188], [0.073,0.322], [0.252,0.490]⟩	⟨[0.188,0.252], [0.128,0.490], [0.073,0.400]⟩	⟨[0.322,0.400], [0.252,0.490], [0.128,0.188]⟩	⟨[0.188,0.490], [0.252,0.322], [0.073,0.600]⟩	⟨[0.188,0.400], [0.128,0.322], [0.252,0.322]⟩

Step 4:

Consider the information in Eq. (19) and Eq. (21) with the weight vector $Y_{W(i)} = (0.2,0.1,0.2,0.3,0.2)^T$, the data in Table 2 is converted to the data in Table 3, which is called a weighted aggregated matrix

Table 3. The border approximation area.

MABAC	IVMPFWA	IVMPFWG
$Y^{A_{t(1)}}$	⟨[0.1180,0.1767], [0.0630,0.3088], [0.2399,0.4731]⟩	⟨[0.1177,0.1764], [0.0626,0.3085], [0.2396,0.4727]⟩
$Y^{A_{t(2)}}$	⟨[0.1767,0.2399], [0.1180,0.4731], [0.0630,0.3854]⟩	⟨[0.1764,0.2396], [0.1177,0.4727], [0.0626,0.3851]⟩
$Y^{A_{t(3)}}$	⟨[0.3088,0.3854], [0.2399,0.4731], [0.1180,0.1767]⟩	⟨[0.3085,0.3851], [0.2396,0.4727], [0.1177,0.1764]⟩
$Y^{A_{t(4)}}$	⟨[0.1767,0.4731], [0.2399,0.3088], [0.0630,0.5787]⟩	⟨[0.1764,0.4727], [0.2396,0.3085], [0.0626,0.5782]⟩
$Y^{A_{t(5)}}$	⟨[0.1767,0.3854], [0.1180,0.3088], [0.2399,0.3088]⟩	⟨[0.1764,0.3851], [0.1177,0.3085], [0.2396,0.3085]⟩

Step 5:

Using the technique in Eq. (24) and Eq. (25), this study aims to determine the distance values among the data in Table 2 and Table 3, which is called distance values are available in Tables 4 and Table 5.

Table 4. Distance of each alternative using IVMPFWA.

MABAC	$Y^{A_{t(1)}}$	$Y^{A_{t(2)}}$	$Y^{A_{t(3)}}$	$Y^{A_{t(4)}}$	$Y^{A_{t(5)}}$
$Y^{A'_{t(1)}}$	0.01477	0.0151	0.01579	0.01702	0.01497
$Y^{A'_{t(2)}}$	0.00816	0.00835	0.00873	0.00946	0.00826
$Y^{A'_{t(3)}}$	0.00148	0.00152	0.0016	0.00178	0.00149
$Y^{A'_{t(4)}}$	0.00527	0.00538	0.00562	0.00602	0.00535
$Y^{A'_{t(5)}}$	0.01209	0.01236	0.01293	0.01393	0.01225

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Table 5. Distance of each alternative using IVMPFWG.

MABAC	$Y^{A_{t(1)}}$	$Y^{A_{t(2)}}$	$Y^{A_{t(3)}}$	$Y^{A_{t(4)}}$	$Y^{A_{t(5)}}$
$Y^{A'_{t(1)}}$	0.01446	0.01278	0.01552	0.01667	0.01472
$Y^{A'_{t(2)}}$	0.00785	0.00803	0.00846	0.00911	0.00801
$Y^{A'_{t(3)}}$	0.00117	0.0012	0.00132	0.00143	0.00123
$Y^{A'_{t(4)}}$	0.00558	0.0057	0.0059	0.00637	0.00561
$Y^{A'_{t(5)}}$	0.01244	0.01268	0.0132	0.01429	0.01251

Step 6:

Considering the technique in Eq. (26), the overall utility value is listed in Table 6 by using the data in Tables 4 and 5.

Table 6. Final score of the alternative.

MABAC	IVMPFWA	IVMPFWG
$Y^{A_{t(1)}}$	0.00835	0.00829
$Y^{A_{t(2)}}$	0.00854	0.00848
$Y^{A_{t(3)}}$	0.00893	0.00888
$Y^{A_{t(4)}}$	0.00964	0.00957
$Y^{A_{t(5)}}$	0.00847	0.00842

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Step 7:

Compute the ranking values by using the data in Table 6 to find the best and worst decisions for both operators in the MABAC model, which is described in Table 7.

Table 7. Final ranking result of the alternatives.

MABAC	Ranking
IVMPFWA	$Y^{A_{t(3)}} > Y^{A_{t(4)}} > Y^{A_{t(2)}} > Y^{A_{t(5)}} > Y^{A_{t(1)}}$
IVMPFWG	$Y^{A_{t(3)}} > Y^{A_{t(4)}} > Y^{A_{t(2)}} > Y^{A_{t(5)}} > Y^{A_{t(1)}}$

The ranking result shown in Table 7 clearly demonstrates that by applying the proposed MABAC model on the expert preferences located in Table 5, we got $Y^{A_{t(3)}}$ as the best alternative from the selected set of alternatives. It means that based on the given expert information, **LLaMA (Meta)** achieved the highest overall performance, which means **LLaMA (Meta)** demonstrates a lower hallucination risk and higher trustworthiness compared to other alternatives.

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5. Comparative Analysis

In this section, the comparative evaluation between proposed frameworks and existing frameworks is elaborated to show the validity and supremacy of the derived models. The comparative assessment is one of the most valuable procedures to describe the effectiveness of the invented techniques. This section is specially designed for the comparison of the relative performance of the proposed models with some related existing models. For the valuation of the comparative analysis between the proposed and existing techniques, the following existing models are arranged. For instance, Mezei and Wikstrom [33] constructed a novel aggregation approach by integrating the notations of IVFS and ordered weighting averaging aggregation operators. Bustince et al. [34] deliberated the Choquet integrals for IVFSs. Further, Waseem et al. [35] derived the operators based on Hamacher norms for m-PFSs. Moreover, Akram et al. [36] examined the operators for m-PFSs based on Dombi norms. Additionally, Jana and Pal [37] presented the decision-making models for some m-PFSs. Further, Maity and Pal [38] stated the power operators based on Dombi norms for m-PFSs. Moreover, Ali et al. [39] derived the operators for m-PFSs based on Schweizer-Sklar norms. Many decision-making procedures are developed for the interpretation of complex problems. Inspired by these decision-making models, the hybrid decision-making model was invented by Pamucar and Cirovic [40], which is called the multi-attribute border approximation area comparison (MABAC) network. The MABAC model is very efficient in coping with complex data. Further, Akram et al. [41] derived another hybrid model of decision-making technique, called the PROMETHEE technique for m-PFSs. Moreover, Jagtap and Karande [42] invented the technique for order performance by similarity to the ideal solution (TOPSIS) model for m-PFSs. Additionally, Akram and Adeel [43] deliberated the ELECTRE-I and ELECTRE-II models for m-PFSs. Thus, using the data in Table 1, the comparative analysis is mentioned and deliberated in Table 11.

Table 8. Numerical comparative analysis.

Ideas		Score					Ranking
Mezei and Wikstrom [33]		Failed (limitations)					Failed (limitations)
Bustince et al. [34]		Failed (limitations)					Failed (limitations)
Waseem et al. [35]		Failed (limitations)					Failed (limitations)
Akram et al. [36]		Failed (limitations)					Failed (limitations)
Jana and Pal [37]		Failed (limitations)					Failed (limitations)
Maity and Pal [38]		Failed (limitations)					Failed (limitations)
Ali et al. [39]		Failed (limitations)					Failed (limitations)
Pamucar and Cirovic [40]		Failed (limitations)					Failed (limitations)
Akram et al. [41]		Failed (limitations)					Failed (limitations)
Jagtap and Karande [42]		Failed (limitations)					Failed (limitations)
Akram and Adeel [43]		Failed (limitations)					Failed (limitations)
MADM	IVMPFWA	$\Upsilon^{A_{t(1)}}$	$\Upsilon^{A_{t(2)}}$	$\Upsilon^{A_{t(3)}}$	$\Upsilon^{A_{t(4)}}$	$\Upsilon^{A_{t(5)}}$	$\Upsilon^{A_{t(4)}} > \Upsilon^{A_{t(3)}} > \Upsilon^{A_{t(5)}} > \Upsilon^{A_{t(2)}} > \Upsilon^{A_{t(1)}}$
		0.3889	0.4055	0.4722	0.4889	0.4389	

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	IVMPFWG	0.3883	0.4050	0.4718	0.4884	0.4384	$Y^{A_{t(4)}} > Y^{A_{t(3)}} > Y^{A_{t(5)}} > Y^{A_{t(2)}} > Y^{A_{t(1)}}$
MABAC	IVMPFWA	$Y^{A_{t(1)}}$	$Y^{A_{t(2)}}$	$Y^{A_{t(3)}}$	$Y^{A_{t(4)}}$	$Y^{A_{t(5)}}$	$Y^{A_{t(3)}} > Y^{A_{t(4)}} > Y^{A_{t(2)}} > Y^{A_{t(5)}} > Y^{A_{t(1)}}$
		0.00835	0.00854	0.00893	0.00964	0.00847	
	IVMPFWG	0.00829	0.00848	0.00888	0.00957	0.00842	$Y^{A_{t(3)}} > Y^{A_{t(4)}} > Y^{A_{t(2)}} > Y^{A_{t(5)}} > Y^{A_{t(1)}}$

In Table 11, it is clearly shown that the existing ideas are not able to handle the given expert preference located in Table 1, while both the proposed models effectively handle them because of the following reasons.

1) The idea presented by Mezei and Wikstrom [33] is based on interval-valued fuzzy information, while the given expert preference is expressed in interval-valued m-polar fuzzy form, which is a generalization of the existing m-polar fuzzy. So, because of its limited flexibility, it is not able to address the given expert preference. Similarly, the studies presented by Bustince et al. [34], Waseem et al. [35], Akram et al. [36], Jana and Pal [37], Maity and Pal [38], and Ali et al. [39] are also based on limited fuzzy extension, which makes them limited to address the given expert preferences.
2) The MADM model presented by Pamucar and Cirovic [40] is limited to handling the given expert information because it is based on crisp information, while the expert preference is shown in an extended fuzzy form. So, the limited extension of the MABAC approach makes it limited to handling the given expert preferences. Similarly, the other selected MADM models are also limited to handling the given expert judgement because of their limited generalized structure.
3) The proposed MABAC and MADM models are developed under an interval-valued m-polar fuzzy environment, which is a generalization of the existing models. Because of its flexible and generalized structure, it effectively handles the given expert information and gives an accurate final ranking result. Both models give the same ranking result, which shows the accuracy and reliability of the proposed models.

The sensitivity analysis for the proposed model based on the data in Table 1 are listed in Table 12 for different values of the weight vectors.

Table 9. Representation of sensitivity analysis.

Weighted (IVMPFWA)	Score values	Ranking values
0.2,0.1,0.2,0.3, 0.2	0.00835,0.00854,0.00893,0.00964,0.00847	$Y^{A_{t(3)}} > Y^{A_{t(4)}} > Y^{A_{t(2)}} > Y^{A_{t(5)}} > Y^{A_{t(1)}}$
0.2,0.2,0.2,0.2, 0.2	0.00809,0.00827,0.00865,0.00933,0.00819	$Y^{A_{t(4)}} > Y^{A_{t(3)}} > Y^{A_{t(2)}} > Y^{A_{t(5)}} > Y^{A_{t(1)}}$

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0.1,0.1,0.3,0.4, 0.1	0.00848,0.00867,0.00907,0.00978,0.00859	$Y^{A_{t(4)}} > Y^{A_{t(3)}} > Y^{A_{t(2)}} > Y^{A_{t(5)}} > Y^{A_{t(1)}}$
0.5,0.1,0.2,0.1, 0.1	0.0091,0.0093,0.00973,0.01047,0.00923	$Y^{A_{t(4)}} > Y^{A_{t(3)}} > Y^{A_{t(2)}} > Y^{A_{t(5)}} > Y^{A_{t(1)}}$
0.25,0.25,0.3,0.1,0.1	0.00864,0.00883,0.00923,0.00994,0.00876	$Y^{A_{t(4)}} > Y^{A_{t(3)}} > Y^{A_{t(2)}} > Y^{A_{t(5)}} > Y^{A_{t(1)}}$

According to the technique of proposed model, the best decision is $Y^{A_{t(3)}}$ for the value of weight vector (0.2,0.1,0.2,0.3,0.2), but for the remaining weight vectors, the ranking values same, where the best decision is $Y^{A_{t(4)}}$. After all, the proposed technique is novel and up to date; no one can work on it, so the existing techniques are not able to evaluate the data in Table 1. Hence, the invented approaches are new and valuable to cope with vague and uncertain data in complex scenarios.

6. Conclusion

The IVMPF framework provides an effective platform for the expert to express their preference, even if it is complex. The proposed aggregation operators effectively aggregate the expert information into a single value and help the decision makers to aggregate the complex and uncertain information. In this research work, we invented the following concepts. The main purpose of this manuscript is to evaluate the best and worst decisions among the decisions of Gemini (Google DeepMind), Copilot (GitHub/Microsoft), LLaMA (Meta), Claude (Anthropic), and ChatGPT (OpenAI) based on the following criteria, such as data privacy, Explainability, Hallucination risk, reliability, and bias control. Therefore, the system of the IVMPF-MABAC technique for algebraic information is developed. The invented model is very famous and reliable for coping with awkward and complex information. Therefore, for the valuation or the construction of the derived technique, this study also developed the novel technique of IVMPF information and also investigated their operations based on algebraic norms for the construction of the averaging operator and geometric operator. Lastly, this study worked on the analysis of the numerical examples and the implementation of the comparative analysis among proposed and prevailing frameworks to derive the validity and effectiveness of the introduced approaches.

In the future, our target is to invent the notation of Dombi aggregation operators, Hamacher aggregation operators, Einstein aggregation operators, and Aczel-Alsina aggregation operators under an IVMPF environment. We are also targeting to extend the current notation of WASPAS model, CoCoSo model, TOPSIS model, and DEMATAL model to an interval-valued m-polar fuzzy domain.

Data availability: All data are included in the paper (calculation data is available on request).

Competing interest: We have no competing interest.

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