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A SUPPLEMENT OF CLOSED SUBSETS OF A HYPERGROUP

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Abstract. This study focuses on $\Phi(H)$-supplement of a closed subset of hypergroup H. It is shown that, under certain conditions, closed subsets of a hypergroup included in a closed subgroup have $\Phi(H)$-supplement and having such a supplement for a closed subset is hereditary for quotient hypergroups. In addition, the closed subset generated by the closed subsets with $\Phi(H)$-supplement has $\Phi(H)$-supplement in hypergroup H.

1. Introduction

Hypergroup theory can be considered a useful generalization of group theory in which the combination of two elements is a set rather than an element. The idea was first released by Marty in [6]. In his pioneering work, a *hypergroup* is defined as follows:

We call a set H equipped with a hypermultiplication (while $P(H)$ is the power set of H); $\cdot : H \times H \to P(H)$ is called a *hypergroup* if the following conditions are satisfied:

1. $a \cdot (b \cdot c) = (a \cdot b) \cdot c$ for every $a, b, c \in H$
2. There exists $e \in H$ such that $h \cdot e = \{h\}$ for each $h \in H$.
3. There exists $m^*, k^* \in H$ provided that while $k \in l \cdot m$ then $l \in k \cdot m^*$ and $m \in k^* \cdot l$ for every $k, l, m \in H$.

Here, the element e is called *neutral* which satisfies $h \cdot e = \{h\}$ for each $h \in H$, will be denoted as 1 and it is shown in [10] that the neutral element and h^* for every h defined above are unique in a hypergroup. Due to the uniformity of h^*, there is a function on a hypergroup H as $h \to h^*$ and this function is called an inverse function of H.

Some basic structural facts used in this work, such as isomorphism theorems, modular law, etc., are given in [10], [9] and [7]. In addition to that, when it came to the adoption of normality in hypergroup theory, researchers did this gradually. They first came up with the concept of closed subset and then

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gave a definition of normal subhypergroup.They also obtained some powerful structures such as closed, ultraclosed, invertible, conjugable subhypergroups (see [3] for details). The reader is also referred to the books [3], [2] and [4] for further information.
This study generally uses closed subsets. We can move on to define *closed subset* of a hypergroup using the definition of inverse function given above.

Lemma 1.1. *[10, Lemma 2.1]*
Let H be a hypergroup and $h \in H$. Then the followings are provided:

1. $1 \in h^*h$
2. $h^{**} = h$

Lemma 1.2. *[5, Lemma 3.1]*
Let H be a hypergroup and $A, B \subseteq H$. Then the following are valid:

1. *If $A \subseteq B$ then $A^* \subseteq B^*$*
2. $(AB)^* = B^*A^*$

Here $A^* = \{a^*|a \in A\}$.

We call a subset A of a hypergroup *closed* if $A^*A \subseteq A$.

Lemma 1.3. *[5, Lemma 3.3] A subset A of a hypergroup is closed if and only if $1 \in A$, $A^* = A$ and $AA = A$.*

Lemma 1.4. *[10, Lemma 2.8]*
Let H be a hypergroup and F be a closed subset of H. Then $\{hF|h \in H\}$ is a partition of H.

A closed subset F of a hypergroup H is called strongly normal in H if for each $h \in H$, $h^*Fh \subseteq F$, also for every $h \in H$ we set $h^F := FhF$ and by these sets we define $A//F := \{a^F|a \in A\}$. The hypermultiplication defined on $H//F$ as $a^Fb^F = \{c^F|c \in aFb\}$ for all $a, b \in H$ and is said to be the hypermultiplication on $H//F$ defined by F. By [9] $H//F$ is a hypergroup with hypermultiplication on $H//F$ defined by F. Here, the inverse function is determined as $h^F \to (h^*)^F$. From now on, we call $H//F$ the quotient of H over F.

Here, the inferences obtained from the concept of Frattini- supplement given in [1] are adapted to hypergroups. Consequently, first, $\Phi(H)$ of hypergroup H will be created as the Frattini subgroup equivalent in group theory through the definition of maximal closed subset, and then hypergroup versions of some of its properties that is used frequently will be given. Then, by giving a definition of $\Phi(H) - supplement$ for a closed subset of the hypergroup H, the existence of such a supplement, its minimality, its inheritance for the closed subset contained by the closed subset and for the quotient hypergroup having $\Phi(H) - supplement$ will be investigated. We will also focus on the conditions to have $\Phi(H) - supplement$ for a closed subset generated by closed subsets.

In order for the appearance, we begin by presenting the basic structure of closed subsets from [10],[5], [3], [11] and [7] in the Preliminaries Section. In the following section, the theorems and results aimed at the study are listed. Also, an example is given after the definition of the main concept of the work.

2. Preliminaries

In [11] a description of maximality of a closed subset is given. A closed subset M of a hypergroup H is called maximal in H if $M \neq H$ and while $M \subseteq K \subset H$ for some closed subset K of H then $M = K$.
Since mentioned before, in this work we study on closed subsets, and as in the definition of Frattini subgroup, we need to see that the intersection of maximal closed subsets is again a closed subset.
Another useful tool of Frattini Subgroup of a group is being the set of all non-generators of the group. Here we need such a tool for the remaining part of this study. So we begin with the concept of closed subset generated by a subset. Let X be a subset of a hypergroup H. Then the intersection of all closed subsets of H containing X is a closed subset according to Lemma (2.1). Here we denote this closed subset as $< X >$ and call it the closed subset generated by X. If $X, Y \subseteq H$ then $< X, Y >=< X \cup Y >$. If X and Y are both closed subsets then it is easily seen that $< X \cup Y >= XY = YX$ by Lemma (2.2) and Lemma (1.3). We call an element $x \in H$ of a hypergroup H a $non-generator$ of H if $H =< x, Y >$ implies $H =< Y >$ for any $Y \subseteq H$.

First of all, in the following we see that the intersection of closed subsets of a hypergroup is a closed subset.

Lemma 2.1. *[5, Lemma 3.6] Let $\mathcal{F}$ be a non-empty set of closed subsets of a hypergroup H and $\mathcal{A}$ be the intersection of the closed subsets belong to $\mathcal{F}$, then*

1. *$\mathcal{A}$ is a closed subset of H.*
2. *$\mathcal{A}$ is a strongly normal in H if all subsets in $\mathcal{F}$ are strongly normal in H.*

When the set $\mathcal{F}$ is the set of all the maximal closed subsets of the hypergroup H we directly conclude that $\mathcal{F}$ is closed by the Lemma (2.1). In this paper, the intersection of maximal closed subsets of the hypergroup H will be denoted as $\Phi(H)$.

Lemma 2.2. *[5, Lemma 3.4]*
Let D and E be closed subsets of a hypergroup H. Then DE is a closed subset of H if and only if $DE = ED$.

Definition 2.3. *[3] Let $(H, \circ)$ and $(K, *)$ be two semihypergroups. $f : H \to K$ is called a homomorphism or inclusion homomorphism if for all $x, y \in H$ $f(x \circ y) \subseteq f(x) * f(y)$. It is called a homomorphism good (or strong) while*

$f(x \circ y) = f(x) * f(y)$ and is called an isomorphism if it is one-to-one and onto good homomorphism.

Theorem 2.4. *[8, Theorem 3.3] Let $f : H \to H'$ be a homomorphism, X a non-empty subset of H and $< X >$ is a closed subset generated by X. Then*

1. $f(< X >) \subseteq < f(X) >$
2. $< f(X) > = < f(< X >) >$
3. *If $f(< X >)$ is a closed subhypergroup of H' then $< f(X) > = f(< X >)$.*

In [10] normality is defined in a hypergroup as follows.

Definition 2.5. *Let H be a hypergroup and let D and E be closed subsets of H. For each $d \in D$ if $Ed \subseteq dE$ then we say that D normalizes E. Also, if H normalizes a closed subset F of H, then F is said to be normal in H.*

By this definition of normality we have some useful properties given in the following lemma. Then an adaptation of the modular law for hypergroups is presented.

Lemma 2.6. *[10, Lemma 3.1] Let H be a hypergroup and D and E be closed subsets of H. Assume D normalizes E. Then the following holds.*

1. *$Ed = dE$ for each $d \in D$.*
2. *ED is closed subset of H.*
3. *E is normal in ED.*
4. *$E \cap D$ is closed subset of H and $E \cap D$ is normal in D.*

Lemma 2.7. *[10, Lemma 2.7] Let H be a hypergroup and F be a closed subset of H and $A, B \subseteq H$. Then the following holds.*

1. *If $A \subseteq F$ then $A(B \cap F) = AB \cap F$.*
2. *If $B \subseteq F$ then $(A \cap F)B = AB \cap F$.*

3. Main Results

This section generally includes the results derived from the definition of $\Phi(H) - supplement$. But first of all we begin by giving some basic properties which are adaptation of Frattini subgroup of a group to closed subsets of hypergroup.

Proposition 3.1. *$\Phi(H)$ consists of non-generator elements of the hypergroup H.*

Proof. First, assume x is a non-generator of the hypergroup H and $x \notin \Phi(H)$. Then $x \notin M$ for some maximal closed subset M of H. Since M is maximal and $x \notin M$ then $M << x, M >$ implies $< x, M > = H$ and $M = H$ which is a contradiction. So $x \in \Phi(H)$. Conversely, let $x \in \Phi(H)$ and while $H =< x, Y >$ then $H \neq < Y >$ for some $Y \subseteq H$. Now we conclude that $x \notin Y$ and Y is not a maximal closed subset. There exists some maximal

closed subset M of H such that $< x, Y >< M$ and $H \subseteq M$, which leads to another contradiction. We have x a non-generator element in H. □

Theorem 3.2. *$\Phi(H) = \{1\}$ for a hypergroup H if and only if for every closed subset $\{1\} \neq N$ of H, there exists $S \subset H$ such $H = NS$.*

Proof. Assume that there is no such closed subset S of H as $H = NS$. Then for every maximal closed subset M of H we have $H \neq NM$ and since $M \subseteq NM \subset H$ then $M = NM$ which implies $N \subseteq M$. Hence, the contradiction $N \subseteq \Phi(H)$ is obtained.
On the contrary, assume that there is a closed subset $S \subset H$ that $H = NS$ and $\Phi(H) \neq \{1\}$. Then for $1 \neq a \in \Phi(H)$, $< a >$ is a closed subset and there exists a closed subset $S \subset H$ that $H =< a > S$. By Proposition (3.1) we get the contradiction $H =< a > S =< a, S >= S$. So $\Phi(H) = \{1\}$. □

Theorem 3.3. *Let K and T be closed subsets of a hypergroup H and $K \subseteq \Phi(T)$. Then $K \subseteq \Phi(H)$.*

Proof. Assume $K \nsubseteq \Phi(T)$. Then $K \nsubseteq M$ for some maximal closed subset M of H. Now $H =< K \cup M >= KM$ and $T = KM \cap T$. By Lemma (2.7) we write

$$T = KM \cap T = K(M \cap T) =< K, M \cap T >.$$

Since $K \subseteq \Phi(T)$ with Proposition (3.1) $T = M \cap T \Rightarrow T \subseteq M$ and contradicts $K \nsubseteq M$. □

Definition 3.4. *Let H be a hypergroup, N be a closed subset of H and S be a closed subset of H such that N normalizes S. The S is called a $\Phi(H) -$ supplement of N in H if $H = NS$ and $N \cap S \subseteq \Phi(S)$. If for every closed subset N of H has $\Phi(H) -$ supplement in H, then H is said to be $\Phi -$ supplemented.*

Example 3.5. *Consider the abelian group $(\mathbb{Z}_6, +)$ and let ρ be the equivalence relation mentioned in [3], with classes $\overline{x} = \{x, -x\}$. If hyperoperation on $\mathbb{Z}_6$ is defined as $\overline{x} \circ \overline{y} = \{\overline{x+y}, \overline{x-y}\}$, then $(\mathbb{Z}_6/\rho, \circ)$ is a hypergroup. The table derived with this operation is given as:*

$\circ$	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	$\bar{5}$
$\bar{0}$	$\{\bar{0}\}$	$\{\bar{1},\bar{5}\}$	$\{\bar{2},\bar{4}\}$	$\{\bar{3}\}$	$\{\bar{2},\bar{4}\}$	$\{\bar{1},\bar{5}\}$
$\bar{1}$	$\{\bar{1}\}$	$\{\bar{0},\bar{2}\}$	$\{\bar{3},\bar{5}\}$	$\{\bar{4}\}$	$\{\bar{3},\bar{5}\}$	$\{\bar{0},\bar{2}\}$
$\bar{2}$	$\{\bar{2}\}$	$\{\bar{1},\bar{3}\}$	$\{\bar{0},\bar{4}\}$	$\{\bar{5}\}$	$\{\bar{0},\bar{4}\}$	$\{\bar{1},\bar{3}\}$
$\bar{3}$	$\{\bar{3}\}$	$\{\bar{2},\bar{4}\}$	$\{\bar{1},\bar{5}\}$	$\{\bar{0}\}$	$\{\bar{1},\bar{5}\}$	$\{\bar{2},\bar{4}\}$
$\bar{4}$	$\{\bar{4}\}$	$\{\bar{3},\bar{5}\}$	$\{\bar{0},\bar{2}\}$	$\{\bar{1}\}$	$\{\bar{0},\bar{2}\}$	$\{\bar{3},\bar{5}\}$
$\bar{5}$	$\{\bar{5}\}$	$\{\bar{0},\bar{4}\}$	$\{\bar{1},\bar{3}\}$	$\{\bar{2}\}$	$\{\bar{1},\bar{3}\}$	$\{\bar{0},\bar{4}\}$

There are two closed subsets of $(\mathbb{Z}_6/\rho, \circ)$: $M = \{\bar{0}, \bar{2}, \bar{4}\}$ and $N = \{\bar{0}, \bar{3}\}$. In this case $\Phi(\mathbb{Z}_6/\rho) = \{\bar{0}\}$. One can easily check that M and N are $\Phi(\mathbb{Z}_6/\rho)$-supplements to each other in $\mathbb{Z}_6/\rho$.

Lemma 3.6. *Let H be a hypergroup, N be a closed subset of H. If S is $\Phi(H)-$supplement of N in H, then S is a minimal supplement of N in H.*

Proof. Since S is $\Phi(H)-$*supplement* of N in H, then $H = NS$ and $N \cap S \subseteq \Phi(S)$. Let K be a $\Phi(H)-$*supplement* of N in H and $K \subseteq S$. Hence $H = NK$. Since $N \cap S$ is closed in S by Lemma (2.6) we have $S = S \cap H = S \cap NK = K(N \cap S)$ from Lemma (2.7) and since K and $N \cap S$ are closed subsets $K(N \cap S) =< K, N \cap S >$ and $N \cap S \subseteq \Phi(S)$ we have $S = K$. □

Theorem 3.7. *Let H be a hypergroup, N be a closed subset of H and $H = NS$ for some $S \subseteq H$. Then, S is minimal if and only if S is $\Phi(H)-$supplement of N in H.*

Proof. First, assume S is minimal and is not a $\Phi(H)-$*supplement* of N in H. Then $N \cap S \nsubseteq \Phi(S)$ and $N \cap S \nsubseteq M$ for some maximal closed subset M of S. So we have $M \subset (N \cap S)M \subseteq S$ which implies

$$S = (N \cap S)M = NM \cap S$$

by Lemma (2.7) and $S \subseteq NM$. Since N is a closed subset, $H = NS \subseteq NM$ implies $H = NM$ which contradicts the minimality of S. On the contrary, S is minimal according to Lemma (3.6). □

Theorem 3.8. *Let H be a hypergroup, N be a closed subset of H and S be a $\Phi(H)-$supplement of N in H. If T is a closed subset of H providing $N \cap S = N \cap T$ and $S \subseteq T$ then T is also a $\Phi(H)-$supplement of N in H.*

Proof. Since S is a $\Phi(H)-$*supplement* of N in H we write $H = NS$ and $N \cap S \subseteq \Phi(S)$. Easily $H = NT$ is provided and by the equility $N \cap S = N \cap T$ we have $N \cap T \subseteq \Phi(S)$. Since $N \cap T$ and S are closed subsets of T by Theorem (3.3) $N \cap T \subseteq \Phi(T)$ and T is a $\Phi(H)-$*supplement* of N in H as desired. □

Theorem 3.9. *Let H be a hypergroup, N be a closed subset of H and S be a $\Phi(H)-$supplement of N in H and let T_i be closed subsets satisfying the conditions in Theorem* (3.8). *Then $< \bigcup T_i >$ is also a $\Phi(H)-$supplement of N in H.*

Proof. The proof is clear when $T :=< \bigcup T_i >$ in Theorem (3.8). □

Theorem 3.10. *If H itself is one of the $\Phi(H)-$supplement of the closed subset N, then H is the only $\Phi(H)-$supplement of N in H.*

Proof. Let S be a $\Phi(H)-$*supplement* of N in H, then $H = NS$ and $N \cap S \subseteq \Phi(S)$. Since H is also $\Phi(H)-$*supplement* of N in H we have

$$N = N \cap H \subseteq \Phi(H).$$

Also

$$H = NS =< N, S > \text{ and } N \subseteq \Phi(H)$$

implies $H =< S >= S$ by Proposition (3.1). □

Corollary 3.11. *Let K be a closed subset of a hypergroup H such that $K \subseteq \Phi(T)$ for some closed subset T of H. Then H is the only $\Phi(H) - supplement$ of K in H.*

Proof. Obviously $H = HK$ and by Theorem (3.3), $K = K \cap H \subseteq \Phi(H)$ and we can easily conclude that H is the only $\Phi(H) - supplement$ of K in H with the help of Theorem (3.10). □

Corollary 3.12. *H is the only $\Phi(H) - supplement$ of $\Phi(H)$ in H.*

Proof. Since $\Phi(H)$ is a closed subset of H according to Lemma (2.6), we can consider K to be $\Phi(H)$ in Corollary (3.11), then the proof ends. □

Lemma 3.13. *Let $\varphi : H \to K$ be a hypergroup isomorphism of hypergroups H, K. Then $\varphi(\Phi(H)) \subseteq \Phi(K)$.*

Proof. Let $\varphi(a) \in \varphi(\Phi(H))$ for some $a \in \Phi(H)$. Then a is a non-generator element of H. We will now show that $\varphi(a)$ is a non-generator element of K. Let $K =< \varphi(a), X >$ for some subset X of K. Since φ is an isomorphism $K = \varphi(H)$ and we have $\varphi{-}1(X) \subseteq H$. By Theorem (2.4)

$$K =< \varphi(a), X > \Rightarrow \varphi^{-1}(K) = \varphi^{-1}(< \varphi(a), X >) \Rightarrow H =< a, \varphi^{-1}(X) >$$

which implies

$$H =< \varphi^{-1}(X) > \Rightarrow K =< X >$$

and $\varphi(a)$ is a non-generator of K and $\varphi(\Phi(H)) \subseteq \Phi(K)$. □

Theorem 3.14. *Let H be a hypergroup and N be a closed subset of H. Then $\Phi(H)N//N \subseteq \Phi(H//N)$.*

Proof. When $\varphi = \mathrm{I}_{H//N}$ in Lemma (3.13), the result follows. □

Theorem 3.15. *Let N be a closed subset of a hypergroup H and S be $\Phi(H) - supplement$ of N in H. If K is a closed subset of H such that $K \subseteq S$ then $S//K$ is $\Phi(H//K) - supplement$ of $N//K$ in $H//K$.*

Proof. Since S is $\Phi(H) - supplement$ of N in H, then $H = NS$ and $N \cap S \subseteq \Phi(S)$. One can easily see that

$$H//K = (N//K)(S//K)$$

and

$$(N//K) \cap (S//K) \subseteq (N \cap S)//K \subseteq \Phi(S)//K.$$

By Theorem (3.14) $\Phi(S)//K \subseteq \Phi(S//K)$. □

Theorem 3.16. *Let H be a hypergroup and N be a closed subset of H, S be $\Phi(H)-$supplement of N in H and K be a closed subset of H such that $S \subseteq K$ Then $N \cap K$ has $\Phi(H)-$supplement in K.*

Proof. Obviously $N \cap K$ is a closed subset of H. Since S is a $\Phi(H)-$*supplement* of N in H, $H = NS$ and $N \cap S \subseteq \Phi(S)$. So we have $H = NS$, which implies that

$$K = H \cap K = NS \cap K = (N \cap K)S$$

by Lemma (2.7) and

$$(K \cap N) \cap S = K \cap (N \cap S) \subseteq K \cap \Phi(S) \subseteq \Phi(S).$$

Hence $N \cap K$ has $\Phi(H)-$*supplement* in K. □

Theorem 3.17. *Let H be a hypergroup and N be a closed subset of H, S be $\Phi(H)-$supplement of N in H. For a closed subset K of H, if $K \subseteq \phi(H)$ then S is a $\Phi(H)-$supplement of NK in H.*

Proof. $H = NS$ and $N \cap S \subseteq \Phi(S)$ since S is $\Phi(H)-$*supplement* of N in H , so for a closed subset K of H, $H = NKS = (NK)S$. Now we must show that $NK \cap S \subseteq \Phi(S)$. Suppose that $NK \cap S \nsubseteq \Phi(S)$. So $NK \cap S \nsubseteq M$, for some maximal closed subset M of S and then $S = <NK \cap S, M>$. Hence $S = <NK \cap S, M>$ implies

$$H = NS = N<NK \cap S, M> \subseteq N<NK, M> = <K, N, M>.$$

Therefore, $H = <K, N, M>$ and then $H = NM$ since $K \subseteq \Phi(S)$. But we have $S = M$ because of the minimality of S from Lemma (3.6), which is a contradiction. Therefore, $NK \cap S \subseteq \Phi(S)$ and S is a $\Phi(H)-$*supplement* of NK in H. □

4. Conclusion

In this study, the definition of $\Phi(H)-$*supplement* is created and exemplified by using the intersection of closed subsets ($\Phi(H)$) of a hypergroup H. Some useful tools for this special subgroup, similar to the Frattini subgroup in group theory, have also been provided for hypergroups by $\Phi(H)$. These tools have an important place in the study. The relation between the existence of a given closed subset having $\Phi(H)-$*supplement* of a hypergroup and the minimality of this supplement is characterized. It turns out that the closed subset of the hypergroup H that is contained in the closed subset, which has $\Phi(H)-$*supplement* also has $\Phi(H)-$*supplement* under certain conditions. For the existence of $\Phi(H)-$*supplement* in quotient hypergroups, it is first shown that the hypergroup isomorphism preserves the intersection of closed subsets in the range set, and then, using this information, the desired is obtained by means

of a special hypergroup isomorphism. The paper also concludes that the closed subset generated by closed subsets of $\Phi(H)-$*supplement* has the same property.

What is done here for closed subsets can also be done for ultraclosed, invertible, and conjugable subhypergroups. One may find a connection between having $\Phi(H)$-supplement for a given closed subhypergroup N and for a given ultraclosed subhypergroup (invertible, conjugable) N, of a hypergroup H by well known inclusions of these types of subhypergroups.

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